of phosphorus in the residue after diffusion; for the reduction of corrosive sublimate to calomel cannot be otherwise explained. Moreover, the presence of free chlorine in the diffused gases was shown by the reaction with iodide of potassium and starch.

We are continuing this research, and hope to lay before the Society the results of an examination of the most prominent cases of so-called abnormal vapour-density.

IV. "On a Simple Formula and Practical Rule for calculating Heights barometrically without Logarithms." By ALEXANDER J. ELLIS, B.A., F.C.P.S. Communicated by Dr. NEIL ARNOTT, F.R.S. Received February 23, 1863.

The following formula and table for calculating heights barometrically without logarithms will be found to give the same results as Laplace's formula up to 30,000 feet, and the table can be readily extended if required. Let

L degrees be the mean latitude of the two stations,
$l = 2{\cdot}6257 \cos 2\mathrm{L}$, $\mathrm{G} = 1 + 0{\cdot}0026257 \cos 2\mathrm{L}$,
$\mathrm{R} = 20888629$, the number of feet in the earth's radius.

At the lower station.

H feet, its height above the sea, $\mathrm{H}'' = \mathrm{H}^2 \div \mathrm{R}$,
B units of any kind, height of barometer, uncorrected,
B′ " " " " " , corrected,
A deg. Fahr., A′ deg. Cent., A″ deg. Reaum., temperature of air,
M " " , M′ " " , M″ " " , " of mercury.

At the upper station.

h, h'', b, b', a, a', a'', m, m', m'' in the same sense.

Then

$$h - \mathrm{H} = \left[52400\frac{\mathrm{B}-b}{\mathrm{B}+b} + c - 2{\cdot}35\,.\,(\mathrm{M}-m)\right].\frac{836+\mathrm{A}+a}{900}$$
$$+ {\cdot}001\,.\,(h-\mathrm{H})\,l + h'' - \mathrm{H}'', \quad \ldots\ldots \quad (a)$$

where $\mathrm{M}-m=0$, when B, b = B′, b', and

$$2{\cdot}35\,(\mathrm{M}-m) = 4{\cdot}23\,.\,(\mathrm{M}'-m') = 5{\cdot}29\,.\,(\mathrm{M}''-m'')$$
$$\frac{836+\mathrm{A}+a}{900} = \frac{500+\mathrm{A}'+a'}{500} = \frac{400+\mathrm{A}''+a''}{400}.$$

The numbers c, l, h'', H″ are to be taken from the table on the next page, as will appear by the following examples.

Ex. 1. Height of Mont Blanc above Geneva from the observations of MM. Bravais and Martins, August 29th, 1844.

A'	19·3	B	729·65 mm.	M'	18·6	H	1335·33
a'	− 7·6	b	424·05	m'	− 4·2	L	46
	500·0	B+b	1153·70		22·8	l	0·09
	511·7	B−b	305·60		× 4·23		× 14½
				p	96·4	q	1·3

305·6 × 52400 ÷ 1153·7
= 13880·0
272·9 c for 1300
61·6 diff. for 880
$\overline{1}03\cdot6$ $-p$
14118·1

14118·1 × 511·7 ÷ 500
= 14448·5
10·8 h'' for 15000 } h,
1·2 diff. for 800 }
$\overline{1}\cdot9$ $-$H$''$ for 1500, H
$\overline{1}8\cdot7$ $-q$
h−H = 14459·0 difference of level.

Result by Laplace's formula 14459·4.

Table of Corrections.

Feet.	c	Diff. for 100 feet.	h'' + H$''$−	Diff. for 100 feet.	L l+	L l−	l
1000	− 0·3	0·06	0·05	0·01	0°	90°	2·65
2000	+ 0·3	0·20	0·20	0·02	5	85	2·61
3000	2·3	0·41	0·43	0·03	10	80	2·49
4000	6·4	0·72	0·77	0·04	15	75	2·29
5000	13·6	1·08	1·20	0·05	20	70	2·03
6000	24·4	1·54	1·72	0·06	21	69	1·97
7000	39·8	2·07	2·35	0·07	22	68	1·91
8000	60·5	2·68	3·06	0·08	23	67	1·84
9000	87·3	3·35	3·88	0·09	24	66	1·77
10000	120·8	4·16	4·79	0·10	25	65	1·70
11000	162·4	5·04	5·79	0·11	26	64	1·63
12000	212·8	6·01	6·89	0·12	27	63	1·56
13000	272·9	7·07	8·09	0·13	28	62	1·48
14000	343·6	8·27	9·38	0·14	29	61	1·40
15000	426·3	9·55	10·77	0·15	30	60	1·33
16000	521·8	10·98	12·26	0·16	31	59	1·24
17000	631·6	12·40	13·84	0·17	32	58	1·16
18000	755·6	14·35	15·51	0·18	33	57	1·08
19000	899·1	16·05	17·28	0·19	34	56	0·99
20000	1059·6	18·03	19·15	0·20	35	55	0·91
21000	1239·9	20·23	21·11	0·21	36	54	0·82
22000	1442·2	22·56	23·17	0·22	37	53	0·73
23000	1667·8	25·14	25·33	0·23	38	52	0·64
24000	1919·2	27·96	27·58	0·24	39	51	0·55
25000	2198·8	31·00	29·92	0·24	40	50	0·46
26000	2508·8	34·65	32·36	0·25	41	49	0·37
27000	2852·3	37·96	34·90	0·26	42	48	0·27
28000	3231·9	41·99	37·53	0·27	43	47	0·18
29000	3651·8	46·36	40·26	0·28	44	46	0·09
30000	4115·4		43·09		45	45	0·00

Ex. 2. Rush's balloon ascent, September 10th, 1838 (see Meteorological Papers by Admiral FitzRoy, No. 9, p. 19).

A	60	B'	30·496 in.	H	0
a	5	b'	10·830	L	52
	836	$B'+b'$	41·326	l	0·64
	901	$B'-b'$	19·666		$\times$ 27
				q	17·3

$$\begin{array}{rl} 19{\cdot}666\times 52400\div 41{\cdot}326 & \\ =24935{\cdot}8 & \\ 2198{\cdot}8 & c \text{ for } 25000 \\ \bar{1}81{\cdot}4 & \text{diff. for } -65 \\ \hline 27116{\cdot}0 & \end{array} \qquad \begin{array}{rl} 27116\times 901\div 900 & \\ =27146{\cdot}1 & \\ 34{\cdot}9 & h'' \text{ for } 27000 \\ 0{\cdot}3 & \text{diff. for } 100 \\ \bar{1}82{\cdot}7 & -q \\ \hline h-\mathrm{H}=27164{\cdot}0 & \end{array}$$

Laplace's formula gives the same result.

As the British highlands do not exceed 5000 feet in altitude, and lie near the parallel of 56° north latitude, the corrections will nearly destroy each other. The following simple rule will therefore suffice for calculating all British heights :—

"Multiply the difference of the barometers by 524, and divide the product by the sum of the barometers, retaining three decimal places. Multiply this quotient by the sum of the temperatures of the air increased by 836, and divide the product by 9, keeping one decimal place. For aneroid and corrected mercurial barometers, the quotient is the height in English feet. For uncorrected barometers, subtract $2\frac{1}{2}$ times the difference of the temperatures of the mercury."

Ex. 3. Height of Ben Lomond (see Col. Sir H. James's Instructions for taking Meteorological Observations, App.).

A	59·0	B	29·890 in.	M	60·8
a	47·8	b	26·656	m	49·3
	836·0	$B+b$	56·546	$M-m$	11·5
	942·8	$B-b$	3·234		$\times 2\frac{1}{2}$
					28·7

$$3{\cdot}234\times 524\div 56{\cdot}546\times 942{\cdot}8\div 9-28{\cdot}7=3110{\cdot}5=h-\mathrm{H}.$$

The height by Laplace's formula is 3110·8, by levelling 3115·8. The accuracy of the present formula is only intended to be tested by Laplace's, and it will be wrong to at least the same extent.

Very good results may also be obtained by neglecting H'', which is always very small, and transposing the terms h'' and $-2{\cdot}35\,(M-m)$; thus

$$h-H=\left(52400\frac{B-b}{B+b}+c+h''\right).\frac{836+A+a}{900}+{\cdot}001.(h-H)\,l-2\tfrac{1}{2}(M-m),$$

where $2\frac{1}{2}$ is written for $2{\cdot}35$ to compensate for omitting to multiply the latter by $(836+A+a)\div 900$. This approximate form gives rise to the following practical rule for determining heights under 10,000 feet, embodying so much of the Table of corrections as is necessary for that purpose.

"Multiply the difference of the barometers by 52400, and divide by the sum of the barometers. If the number of clear thousands in the quotient be

	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,
add	0,	0·5,	2·7,	7·2,	14·8,	26·1,	42·2,	63·6,	91·2,	125·6
and		0·2,	0·5,	0·8,	1·1,	1·6,	2·1,	2·9,	3·1	

for every additional hundred. Then multiply the result by the sum of the temperatures of the air increased by 836, and divide the product by 900. To this quotient

add for lat.	0,	10,	20,	30,	32,	34,	36,	38,	40,	42,	44,
subtract for lat.	90,	80,	70,	60,	58,	56,	54,	52,	50,	48,	46
the numbers....	2·6,	2·5,	2·0,	1·3,	1·2,	1·0,	0·8,	0·6,	0·5,	0·3,	0·1

for every clear thousand it contains. For aneroid and corrected mercurial barometers this result is the height in English feet. For uncorrected mercurial barometers, subtract $2\frac{1}{2}$ times the difference of the temperatures of the mercury.

"The barometers may be expressed in any units. If the temperatures are expressed in

degrees Centigrade, use........	500,	500,	$4\frac{1}{2}$,
degrees Reaumur, use	400,	400,	$5\frac{1}{2}$,
in place of..................	836,	900,	$2\frac{1}{2}$,

which are only suited for degrees Fahrenheit. The rule and the other numbers remain unaltered, and the result is in English feet."

Ex. 4. Height of Guanaxuato in Mexico.

A	77·5	B	30·046	M	77·5	L	21
a	70·3	b	23·660	m	70·3	l	2·0
	836·0	B+b	53·706	M−m	7·2		× 6·8
	983·8	B−b	6·386		$2\frac{1}{2}$	q	13·6
				p	18·0		

$6{\cdot}386\times52400\div53{\cdot}706$	$6260{\cdot}5\times983{\cdot}8\div900$
$=6230{\cdot}7$	$=6842{\cdot}5$
$26{\cdot}1$ for 6000	$13{\cdot}6 \quad q$
$3{\cdot}7$ diff. for 230	$\bar{1}81{\cdot}0-p$
$6260{\cdot}5$	$h-\mathrm{H}=6838{\cdot}1$

Result by Laplace's formula 6838·2.

These results are obtained by transforming Laplace's formula as follows. The original expression in the Méc. Cél. vol. iv. p. 293, reduced to English measures and the present notation, is

$$\left.\begin{aligned}&h-\mathrm{H}=60158{\cdot}71\,.\,(1+0{\cdot}002845\cos 2\mathrm{L})\,.\,\frac{836+\mathrm{A}+a}{900}\\&\times\left[\left(1+\frac{h-\mathrm{H}}{\mathrm{R}+\mathrm{H}}\right)\right](\log\mathrm{B}'-\log b')+\frac{h-\mathrm{H}}{\mathrm{R}+\mathrm{H}}\,.\,0{\cdot}868589\Big]\end{aligned}\right\}\,.\,.(b)$$

which Delcros has transformed (in 'Annuaire Météorologique de la France' for 1849) to the equivalent of

$$\left.\begin{aligned}&h-\mathrm{H}=60158{\cdot}71\times[\log\mathrm{B}-\log b-0{\cdot}0000389278\,.\,(\mathrm{M}-m)]\\&\times\frac{836+\mathrm{A}+a}{900}\times\mathrm{G}\times\left[1+\frac{h-\mathrm{H}+52251}{\mathrm{R}}+\frac{\mathrm{H}}{\frac{1}{2}\mathrm{R}}\right]\end{aligned}\right\}\,.\,(c)$$

The last factor may be split into the two

$$\left(1+\frac{52251}{\mathrm{R}}\right)\cdot\left(1+\frac{h+\mathrm{H}}{\mathrm{R}}\right)$$

without sensible error. Then, since

$$60158{\cdot}71\times\left(1+\frac{52251}{\mathrm{R}}\right)=60309{\cdot}19$$

and

$$60309{\cdot}19\times0{\cdot}0000389278=2{\cdot}34770,$$

if we put $h-\mathrm{H}$ for the product of the three first factors on the right-hand side in (c), we find

$$\left.\begin{aligned}&h-\mathrm{H}=[60309{\cdot}19\,.\,(\log\mathrm{B}-\log b)-2{\cdot}34770\,.\,(\mathrm{M}-m)]\,.\,\frac{836+\mathrm{A}+a}{900}\\&+\frac{h-\mathrm{H}}{1000}\times2{\cdot}6257\cos 2\mathrm{L}+\frac{h^2-\mathrm{H}^2}{\mathrm{R}}\end{aligned}\right\}\,(d)$$

Putting 2·35 for 2·34770, and l, h'', H'' for their values, this form (d) will be identical with (a), provided that

$$60309{\cdot}19\,.\,(\log\mathrm{B}-\log b)=52400\,.\,\frac{\mathrm{B}-b}{\mathrm{B}+b}+c\quad.\quad.\quad(e)$$

Now putting $B-b=yB$, we have

$$\frac{B-b}{B+b}=\frac{y}{2-y}=\frac{1}{2}\cdot\left(y+\frac{1}{2}y^2+\frac{1}{4}y^3+\frac{1}{8}y^4+\ldots\right)=\frac{1}{2}z,$$

$$\log B-\log b=\log\frac{1}{1-y}=\mu.\left(y+\frac{1}{2}y^2+\frac{1}{3}y^3+\frac{1}{4}y^4+\ldots\right)=\mu(z+d),$$

where μ is the modulus of the tabular logarithms, and

$$d=\frac{1}{12}y^3+\frac{1}{8}y^4\ldots,$$

always a convergent series as y is always a proper fraction, and small when y is small, as it is for moderate heights.

Hence

$$60309{\cdot}19\,.\,(\log B-\log b)=60309{\cdot}19\times 2\mu\frac{B-b}{B+b}+60309{\cdot}19\,.\,\mu d$$
$$=52384\frac{B-b}{B+b}+c'.$$

The constant 52384 has been changed to 52400 to facilitate calculation and to divide the correction for the first two thousand feet, and c' has consequently been altered to c, the tabular values of which were calculated as follows.

Put $$x=52400\frac{B-b}{B+b}=52400\,.\,\frac{y}{2-y},$$

whence

$$y=\frac{2x}{52400+x}. \qquad (f)$$

Then (e) becomes

$$x+c=60309{\cdot}19\,.\,(\log B-\log b)=-60309{\cdot}19\log(1-y)\quad . \quad (g)$$

Make x successively $=1000$, 2000, &c. up to 30,000, and find the corresponding values of y from (f) and c from (g).

As the differences in the values of c are not uniform, slight errors may arise from neglecting second differences in interpolation, but they can scarcely even affect the result by a single unit, and may therefore be safely disregarded. Laplace's formula itself cannot be depended on within much larger limits.

The Table of corrections and transformation of Laplace's formula here given allow of the following simplification in the logarithmic calculation of $h-H$.

$$
\begin{aligned}
\text{Let}\quad \log n &= \log[\log B - \log b - 0{\cdot}00004\,.\,(M-m)] \\
&\quad + 1{\cdot}8261420 + \log(836 + A + a) \\
&= \log[\log B - \log b - 0{\cdot}00007\,.\,(M'-m')] \\
&\quad + 2{\cdot}0814145 + \log(500 + A' + a') \\
&= \log[\log B - \log b - 0{\cdot}00009\,.\,(M''-m'')] \\
&\quad + 2{\cdot}1783245 + \log(400 + A'' + a''),
\end{aligned}
$$

then $$h - H = n + {\cdot}001\,.\,nl + h'' - H'',$$

where $$1{\cdot}8261420 + \log 900 = 2{\cdot}0814145 + \log 500$$
$$= 2{\cdot}1783245 + \log 400 = 4{\cdot}7803845 = \log 60309{\cdot}19.$$

This form requires less previous preparation, avoids the logarithms of numbers near to unity as $\left(1+\frac{A+a-64}{900}\right)$, and allows of the use of foreign data to obtain the result in English feet, so that it only becomes necessary to reduce the height of the lower station to English measures.

V. "Bessel's Hypsometric Tables, as corrected by Plantamour, reduced to English Measures and recalculated." By ALEXANDER J. ELLIS, Esq. Communicated by Dr. NEIL ARNOTT. Received February 23, 1863.

These Tables, with the preliminary explanations respecting their correction and reduction, have been, by direction of the Council, communicated to Admiral FitzRoy for insertion in the "Meteorological Papers published by Authority of the Board of Trade," and will appear in the twelfth Number of the series.

April 16, 1863.

Dr. WILLIAM ALLEN MILLER, Treasurer and Vice-President, in the Chair.

Pursuant to notice given at the last Meeting, the Right Honourable Sir Edmund Walker Head, Baronet, was proposed for election and immediate ballot.

The ballot having been taken, Sir Edmund W. Head was declared duly elected a Fellow of the Society.